Name: ___________________________________ # ___________

Honors Coordinate Algebra: Period _____________
Ms. Pierre
Date: ______________

3.8.2 Geometric Sequences

Warm Up

This time, Julian left his homework in the back pocket of his jeans, and his mom put his jeans through the washing machine. Again he finds himself missing the last piece of each pattern. Describe each pattern and determine the next number in the list.

1. 1, 3, 9, 27, ...

In this list of numbers, each number is 3 times the previous number. Multiply 27 by 3 to get the next number in the list. The next number in the list is 81.

2. 256, 64, 16, 4, ...

In this list of numbers, each number is $\frac{1}{4}$ of the previous number. Multiply 4 by $\frac{1}{4}$ to get the next number in the list. The next number in the list is 1.

3. $4, \frac{8}{3}, \frac{16}{9}, \frac{32}{81}, \ldots$

In this list of numbers, each number is $\frac{2}{3}$ of the previous number. Multiply $\frac{32}{81}$ by $\frac{2}{3}$ to get the next number in the list. The next number in the list is $\frac{64}{243}$.

4. $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots$

In this list of numbers, each number is $-\frac{1}{2}$ of the previous number. Multiply $-\frac{1}{16}$ by $-\frac{1}{2}$ to get the next number in the list. The next number in the list is $\frac{1}{32}$.

Geometric sequences are exponential functions that have a domain of consecutive positive integers. Geometric sequences can be represented by formulas, either explicit or recursive, and those formulas can be used to find a certain term of the sequence or the number of a certain value in the sequence.

Key Concepts

- A geometric sequence is a list of terms separated by a **constant ratio**, the number multiplied by each consecutive term in a geometric sequence.
- A geometric sequence is an exponential function with a domain of positive consecutive integers in which the ratio between any two consecutive terms is equal.
- The rule for a geometric sequence can be expressed either explicitly or recursively.
- The explicit rule for a geometric sequence is $a_n = a_1 \bullet r^{n-1}$, where a_1 is the first term in the sequence, n is the term, r is the constant ratio, and a_n is the nth term in the sequence.
- The recursive rule for a geometric sequence is $a_n = a_{n-1} \bullet r$, where a_n is the nth term in the sequence, a_{n-1} is the previous term, and r is the constant ratio.

Example 1

Find the constant ratio, write the explicit formula, and find the seventh term for the following geometric sequence.

3, 1.5, 0.75, 0.375, ...

1. Find the constant ratio by dividing two successive terms.

   $1.5 \div 3 = 0.5$

2. Confirm that the ratio is the same between all of the terms.

   $0.75 \div 1.5 = 0.5$ and $0.375 \div 0.75 = 0.5$

3. Identify the first term (a_1).

   $a_1 = 3$

4. Write the explicit formula.

   $a_n = a_1 \cdot r^{n-1}$ Explicit formula for any given geometric sequence

   $a_n = (3)(0.5)^{n-1}$ Substitute values for a_1 and n.

5. To find the seventh term, substitute 7 for n.

   $a_7 = (3)(0.5)^{7-1}$

   $a_7 = (3)(0.5)^6$ Simplify.

   $a_7 = 0.046875$ Multiply.

   The seventh term in the sequence is 0.046875.

Example 2

The fifth term of a geometric sequence is 1,792. The constant ratio is 4. Write an explicit formula for the sequence, and then write the corresponding exponential function.

1. The fifth term is 1,792; therefore $n = 5$ and $a_n = 1792$.

2. The constant ratio is 4; therefore, $r = 4$.

3. Substitute the known values into the explicit form of the formula and solve for a.

$1792 = a(4)^{5-1}$	Substitute values.
$1792 = 256a$	Simplify.
$a = 7$	

4. Write the explicit formula.

$a_n = a_1 \bullet r^{n-1}$	Explicit formula for any given geometric sequence
$a_n = 7(4)^{n-1}$	Substitute values.

5. Write the formula in function notation.

$f(x) = 7(4)^{x-1}$

Note that the domain of a geometric sequence is consecutive positive integers.

Example 3

A geometric sequence is defined recursively by $a_n = (a_{n-1})\left(-\frac{1}{3}\right)$, with $a_1 = 729$. Find the first five terms of the sequence, write an explicit formula to represent the sequence, and find the eighth term.

1. Using the recursive formula:

   $a_1 = 729$

   $a_2 = (a_1)\left(-\frac{1}{3}\right)$

   $a_2 = (729)\left(-\frac{1}{3}\right) = -243$

   $a_3 = (-243)\left(-\frac{1}{3}\right) = 81$

   $a_4 = (81)\left(-\frac{1}{3}\right) = -27$

   $a_5 = (-27)\left(-\frac{1}{3}\right) = 9$

   The first five terms of the sequence are 729, –243, 81, –27, and 9.

2. The first term is $a_1 = 729$ and the constant ratio is $r = -\frac{1}{3}$, so the explicit formula is $a_n = (729)\left(-\frac{1}{3}\right)^{n-1}$.

3. Substitute 8 in for n and evaluate.

   $a_8 = (729)\left(-\frac{1}{3}\right)^{8-1}$

   $a_8 = (729)\left(-\frac{1}{3}\right)^{7}$

   $a_8 = -\frac{1}{3}$

   The eighth term of the sequence is $-\frac{1}{3}$.

Guided Practice

Find the constant ratio and write the explicit formula for the *n*th term of each geometric sequence.

1.) 1, 2, 4, 8, 16, ...

$a_n = 2^{n-1}$

2.) $10, -2, \frac{2}{5}, -\frac{2}{25}, \ldots$

$a_n = 10\left(-\frac{1}{5}\right)^{n-1}$

3.) Find the first five terms of the geometric sequence defined as follows:

$a_{n-1} = a_1(3);\ a_1 = -1$

−1, −3, −9, −27, −81

4.) Jade is training for a marathon. During her first week of training, each run she completes is 90 minutes long. She increases the length of each run by 10% each week. Write the explicit formula to represent the length of her run after *n* weeks.

$a_n = 90(1.1)^{n-1}$

Independent Practice

Problem-Based Task 3.8.2: Glass Recycling

The school Recycling Club collected 39.6 pounds of glass on the second day of a collection drive. The members collected 57.024 pounds of glass on the fourth day. The club estimates that the number of pounds collected will form a geometric sequence. Write an explicit formula to represent this scenario. How many pounds of glass, to the nearest tenth of a pound, should the Recycling Club expect to collect on their sixth day of the collection drive?

a. What equation can you write using the information about the second day of the collection drive?

$39.6 = a_1(r)^{2-1}$

$39.6 = a_1 r$

b. What equation can you write using the information about the fourth day of the collection drive?

$57.024 = a_1(r)^{4-1}$

$57.024 = a_1 r^3$

c. How can you use the two equations to find the constant ratio?

Write each equation in terms of a_1.

$$a_1 = \frac{39.6}{r}$$

$$a_1 = \frac{57.024}{r^3}$$

Set the two expressions equal to each other and solve for r.

$$\frac{57.024}{r^3} = \frac{39.6}{r}$$

$$57.024r = 39.6r^3$$

$$r^2 = \frac{57.024}{39.6} = 1.44$$

$$r = 1.2$$

d. What is the explicit formula for the geometric sequence?

Substitute 1.2 for r to find a_1.

$$a_1 = \frac{39.6}{1.2} = 33$$

Substitute the known quantities into the explicit form of the geometric sequence.

$$a_n = 33(1.2)^{n-1}$$

e. Using the rule to estimate, about how many pounds of glass, to the nearest tenth of a pound, should the Recycling Club expect to collect on their sixth day of the collection drive?

Using the formula to estimate, about how many pounds of glass, to the nearest tenth of a pound, should the Recycling Club expect to collect on their sixth day of the collection drive?

$$a_6 = 33(1.2)^{6-1}$$

$$a_6 = 33(1.2)^5$$

$$a_6 = 82.11456$$

To the nearest tenth of a pound, the Recycling Club should expect to collect 82.1 pounds of glass on the sixth day.

Homework

1.) Nigel is participating in a read-a-thon. The number of pages he reads each night follows a geometric sequence. On the second day of the read-a-thon, Nigel read 8 pages. On the fifth day of the read-a-thon, he read 64 pages. Write an explicit formula to represent this scenario.

$a_n = 4(2)^{n-1}$

2.) Mr. Galloway purchased a car for $20,000. The car retains 85% of its value each year. How much will the car be worth in five years?

$10,440.13